Global Trends in Coal Mine Horizontal Stress Measurements

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ABSTRACT

Knowledge of *in situ* stresses is fundamental to many studies in earth sciences, and coal mine ground control is no exception. During the past 20 years, it has become clear that horizontal stress is a critical factor affecting roof stability in underground coal mines. The theory of plate tectonics and the World Stress Map (WSM) project has been extremely helpful in explaining the sources and the orientations of the horizontal stresses observed underground. Recently, WSM geophysicists studying deep-seated stresses have developed a model of how stress magnitudes vary with depth in the crust. They have devoted relatively little attention to near-surface stresses, however. This paper explores the relationships between deep-seated and shallow *in situ* stresses in several of the world's coalfields, using a data base of more than 350 stress measurements from underground coal mines. The analysis indicates that distinct regional trends exist, corresponding roughly to the regional stress fields identified by the WSM. The paper presents equations for estimating stress magnitudes that were developed by treating depth and elastic modulus as independent variables in regression analysis. The magnitude of the horizontal stress increases with depth, at rates that range from 0.8 to 2.0 times the vertical stress, just as the WSM "critically stressed crust" model predicts. Overall, it seems that the stress regimes encountered in underground coal mines are closely linked to those that exist deep in the earth's crust.

ACKNOWLEDGEMENTS

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INTRODUCTION

As early as the 1940's, researchers in British coal mines postulated that large horizontal stresses were responsible for much of the roof damage experienced underground. Philips, (1946) observed that "At depths greater than 700 ft (215 m)…lateral compressive forces cause fracturing along the laminations of the roof beds…the lateral compressive forces increase at a greater rate than the vertical compressive force, and ultimately both forces may be equal." Once rock mechanics researchers began to measure in situ stresses, it became clear that in many cases the horizontal stress actually exceeded the vertical, often by factors of three or more (Dahl and Parsons, 1972; Hoek and Brown, 1980).

A number of theories were proposed to explain the presence of horizontal stress. Two of the earliest were the "Poisson's effect" and the "lithostatic stress state." Both of these theories presumed a static earth, in which the horizontal stresses were generated in response to the vertical overburden load. The "Poisson's effect" model predicted that the horizontal stresses should be about 1/3 of the vertical, while the lithostatic model predicted that the three principal stresses would be approximately equal (McGarr, 1988).

These static earth theories could not explain two key characteristics of horizontal stress:

- Why the horizontal stress often exceeds the vertical (S_V) in magnitude, even at depth, and;
- Why horizontal stresses are typically highly anisotropic, with the major one (S_{Hmax}) significantly larger than the minor (S_{hmin}).

In addition, the Poisson's effect model suffers from severe theoretical errors, because it implicitly assumes that soft sediment (or magma) lithifies in the absence of gravity, and then gravity is instantaneously switched on once the rock has reached an elastic state (McGarr, 1988). As Hoek (2007) writes, the Poisson's effect model was "widely used in the early days of rock mechanics, but proved to be inaccurate and is seldom used today."[1]

[1] The most elegant of the static earth theories is the "spherical shell" model proposed by Sheorey (1994), based in part on earlier work by McCutchen (1982). Sheorey's model considers stresses arising from the geothermal gradient in addition to the Poisson's effect, and it predicts that the horizontal-to-vertical stress ratio may be very large near the surface but declining with depth. Like all static earth theories, however, Sheorey's model can explain neither the near universal anisotropy of the horizontal stress, nor the prevalence of horizontal stresses that are significantly greater than the vertical deep into the crust.

Fortunately, during the 1970's earth scientists were constructing a revolutionary new theory with the breadth and depth to explain all the major tectonic processes observed on the earth. Plate tectonics describes a ***dynamic*** earth, in which the crust of the earth consists of a number of continental plates that are sliding across the softer rock in the mantle below. Where the plates contact each other, their different directions of relative movement create large forces that are transmitted across the plate interiors. The scientists associated with the World Stress Map (WSM) have used a variety of indicators, including earthquake focal mechanisms, wellbore breakouts, and hydraulic fracturing stress measurements, to identify the lithospheric stresses that result from these plate movements. They found that the state of stress is remarkably uniform over vast regions of plate interiors, and that it is due to present-day forces, and not due to residual stresses from past tectonic activity (Zoback and Zoback, 2007). Today there is no argument within the geophyisics community about the general validity of the trends identified by the WSM.

The WSM has found that at any given location, the stress direction within the crust is typically consistent from the "upper 1-3 miles (2-5 km), where essentially all of the wellbore breakout and hydraulic fracturing data come from, down through the lower 3-12 miles (5-20 km), where the majority of crustal earthquakes occur" (Zoback and Zoback, 2002). However, the WSM has specifically excluded "near surface" measurements from its database, because they have sometimes found that there are "marked changes in stress orientations and relative magnitudes with depth in the upper few hundred meters, possibly related to effects of nearby topography or a high degree of surface fracturing" (Zoback, 1992). Later, Zoback and Zoback (2002) stated that "only in situ stress measurements made at depths greater than 100 m are indicative of the tectonic stress field."

Since most underground coal mining takes place within several hundred meters of the surface, it is legitimate to ask how relevant the WSM is to underground coal mining. On the one hand, we do know that topographic features can significantly affect the stresses we observe underground (Molinda et al., 1992; Hasenfus and Su, 2006). On the other hand, since the near-surface is part of the crust, we would certainly expect some relationship to the deep-seated stress patterns. The question is, how closely are the two related? Do we observe the same general trends that the WSM has identified in the deep crust, or are the topographic and other near-surface effects so powerful that they completely mask any relationship?

The first part of the answer to this question was provided in the early 1990's when researchers compiled a data base of stress measurements from U.S. coal mines. The WSM had identified the eastern portion of North America as a stable mid-plate region with a consistent ENE horizontal stress orientation (Zoback and Zoback, 1989). Sure enough, analysis indicated that 75% of the coal mine stress measurement orientations fell within the NE quadrant (Mark, 1991; see figure 1). This finding was particularly meaningful because the data base included measurements made all over the eastern U.S., by a variety of researchers using a number of different techniques. The observed trends were highly significant statistically even though no attempt was made to minimize the effects of bad data by applying a "quality ranking" to the individual measurements. Such quality rankings are normally considered essential for discerning

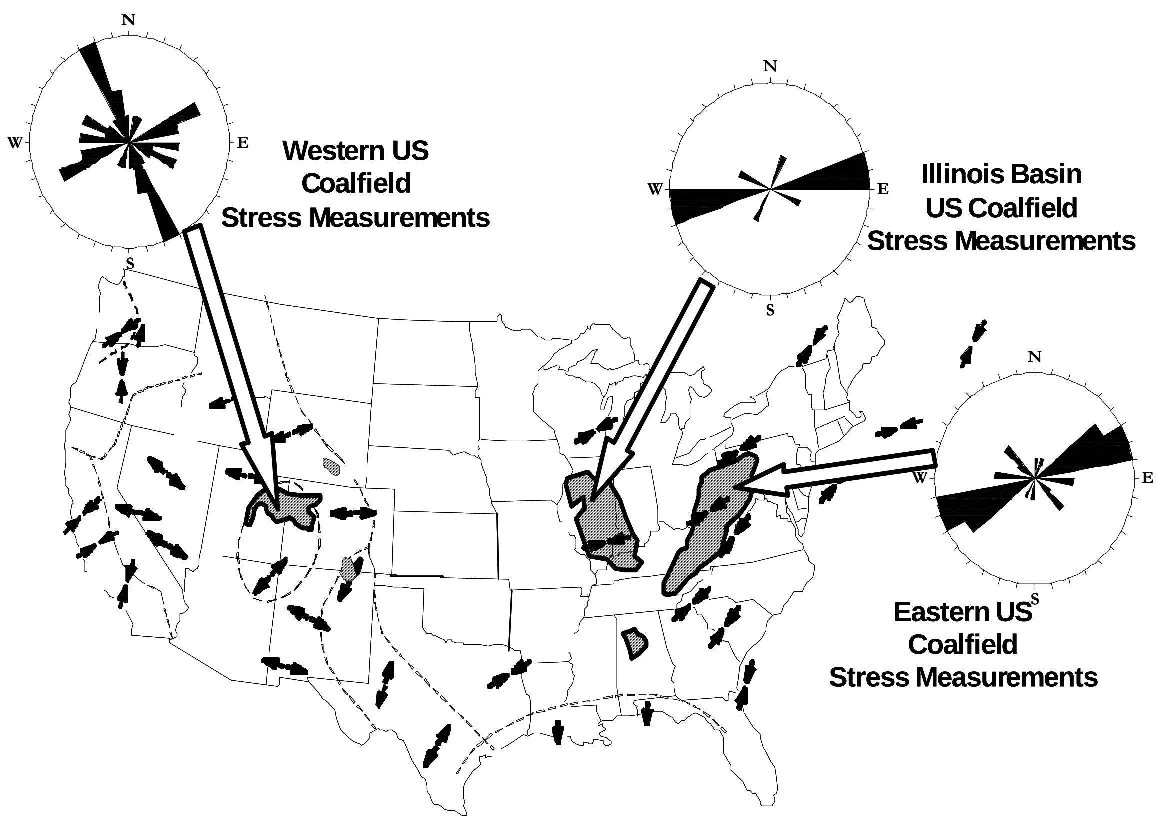

Figure 1. World Stress Map of the United States, compared with stress orientations determined from coalfield stress measurements. The solid arrows show WSM stress direction and the dotted lines delineate stress provinces (Map after Zoback and Zoback, 1989; Coalfield stress measurements from Mark and Mucho, 1994).

underlying trends in stress measurement data (Zoback, 1992; Stacey and Wesseloo, 1998).

The WSM also defined the stress regime within eastern North America as either strike/slip (where the magnitude of the vertical stress falls between the two horizontal stresses) or reverse faulting (where the vertical stress is smaller than both the principal horizontal stresses). Here, the US stress measurement data was in even better agreement, with the maximum horizontal stress exceeding the vertical 97% of the time.

THE WSM AND STRESS ORIENTATIONS IN COALFIELDS AROUND THE WORLD

Stress measurement data bases have now been constructed for a number of the other of the world's coalfields. It makes sense to ask how well the predictions of the WSM compare in those cases.

Western U.S.: According to the WSM, the coalfields of Utah, Colorado, Wyoming, and New Mexico fall within regions of "extension" or normal faulting, where the vertical stress is predicted to be greater than either horizontal stress (see figure 1). In contrast to the eastern "midplate" region, the western regions are seismically active. Stress directions also vary throughout the region.

Mark (1991) analyzed stress measurements from 17 Western U.S. mines, and found no significant regional trends in orientation. The maximum horizontal stress was significantly lower than in the east, and was approximately equal to the vertical stress in most cases.

Germany and the UK: The WSM defines the stress regime in western Europe, including the coalfields of Germany and the UK, as a stable mid-plate area subject to uniform NNW maximum horizontal stress. As in eastern North America, the stress is controlled by plate driving forces acting on the plate boundaries (Muller et al., 1992). Western Europe is considered to be a strike slip environment, with the vertical stress as the intermediate principal stress.

A series of 11 hydrofracturing stress measurements were conducted in four German coal mines between 1989 and 1991 (Muller, 1991). These measurements confirmed that the greatest horizontal stress was oriented NNW. The maximum horizontal stress was reported to exceed the vertical stress down to depths of 4,000 ft (1,200 m).

Cartwright (1997) describes the results from 26 successful overcores conducted by Rock Mechanics Technology (RMT) at 16 mine sites in "virgin or near virgin conditions." The depths of cover ranged from 1,000 to 3,300 ft (300-1,000 m). In every overcore the maximum horizontal stress was located in the NW quadrant, and the vast majority were oriented within a few degrees of NNW (figure 2). The magnitude of the stress was approximately equal to the vertical stress, but there was considerable spread.

Bowen Basin, Queensland, Australia: Australia is considered somewhat unique by the WSM because the stress orientation varies considerably between different regions of the continent, reflecting a variety of plate boundary forces rather than the direction of absolute plate motion (Hillis et al., 1999). In the Bowen Basin coalfields of the central Queensland, the major horizontal stress is consistently oriented NNE, and the vertical

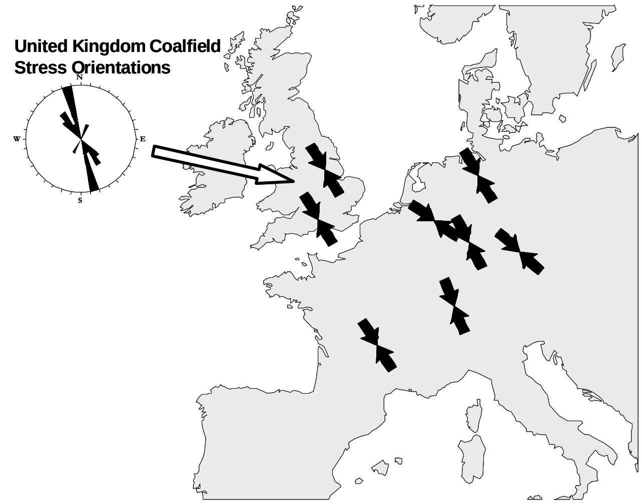

Figure 2. World Stress Map of northern Europe, compared with stress orientations determined from UK coalfield stress measurements (Map after Muller et al. 1992; Stress measurements after Cartwright, 1997).

stress is either the minor or the intermediate principal stress. The region is not seismically active, and evaluation of the available stress measurements found that few were indicative of faults on the verge of movement.

Nemcik et al. (2005) presented the results of 235 measurements of pre-mining stress made by SCT in Australian underground mines. About a third of these measurements were conducted in the Bowen Basin, all at depths of less than 1,000 ft (300 m). Nemcik at al. (2005) reported that "the direction of the major lateral stress was in most cases confined to the N to NE quadrant" (figure 3). The magnitudes of these stresses almost always exceeded the vertical, sometimes by factors of 3 or more.

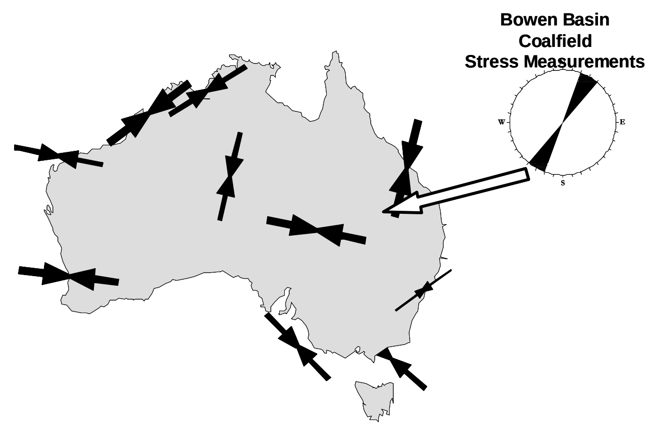

Figure 3. World Stress Map of Australia, compared with stress orientations determined from Bowen Basin coalfield stress measurements (Map after Hillis et al. 1999; Stress measurements after Nemcik et al., 2005).

Sydney Basin, NSW, Australia: The Sydney Basin appears to be the exception that proves the rule. Early studies of horizontal stress in underground mines there found that stress directions could vary widely, even from one section of a mine to another (Gale et al., 1984; Gale, 1986). No consistent regional trend could be observed. As it turns out, the WSM found the same thing

(Hillis et al., 1999). The Sydney Basin is one of Australia's most seismically active, and stress orientations vary widely throughout it. It seems that plate margin effects superimposed on the regional stress direction have resulted in a relatively low anisotropy between the major and minor principal horizontal stresses. As a result, mild perturbations caused by local effects, such as density contrasts, faults, or major geologic structures may cause local stress rotations (Hillis et al., 1999). Moreover, as many as 40% of the stress measurements are indicative of faults at incipient failure. Nemcik et al. (2005) confirmed that even with a very large data base, consisting of approximately 170 stress measurements from mines all over NSW, no consistent trends in stress direction emerged.

Summary: Hillis (1999) concludes that "the apparent consistency between in situ stress measurements and seismicity of the Bowen and Sydney Basins suggests that relatively shallow (1,000-3,300 ft (300-1,000 m)) data … may be representative of the stress at greater, seismogenic depth." Our quick tour around other coalfields of the world leads to the same conclusion, at least with regards to stress orientation and relative magnitude. In every case, the WSM model provided a reasonably accurate prediction of (and explanation for!) the typical stress regime that is observed in underground coal mines. The next question is whether we can also predict the magnitude of the stress using the WSM model.

THE CRITICALLY STRESSED CRUST[2]

The "dynamic earth" plate tectonics model implies that lateral forces are constantly being applied to the brittle upper crust. These forces would continue to build unless there was some mechanism for their release. That mechanism is failure of the crust itself, through faulting. Decades of research along a number of lines of evidence have resulted in what is, in the end, a simple but profound model of the magnitude of the stresses that the crust can carry (Zoback and Zoback, 2002; Zoback and Zoback 2007).

The model begins with Anderson's (1951) classification scheme for relative stress magnitudes in the earth:

- Normal faulting regions, where $S_v > S_{Hmax} > S_{hmin}$ (1)
- Strike slip faulting regions, where $S_{Hmax} > S_v > S_{hmin}$ (2)
- Reverse faulting regions, where $S_{Hmax} > S_{hmin} > S_v$ (3)

Research has shown that the strength of faults can be adequately described by the Coulomb criterion:

$$T = C_o + uS_n \quad (4)$$

where T = the shear strength of the fault plane, C_o = the fault plane's cohesion, u= the friction coefficient, and S_n = the confining stress applied perpendicular to the fault plane.

Using two-dimensional Mohr-Coulomb analysis, the shear stress at failure of an optimally-oriented fault is a function of the difference between the minor (S_3) and the major principal (S_1) stresses (Jaeger and Cook, 1979):

$$(S_1-P_o)/(S_3-P_o) = ((u^2 + 1)^{1/2} + u)^2 \quad (5)$$

where P_o is the pore pressure, and $(S-P_o)$ is the effective stress.

Studies have shown that, at depth, the cohesion is much smaller than the frictional component of the fault strength, the friction coefficient u is typically 0.6-1.0, and the pore pressure is hydrostatic on active faults (Townend and Zoback, 2000). Assuming u=0.6, the following approximate relationships can be derived:

- $S_{Hmax} = 2.3\ S_v$ in reverse faulting regions; (6a)
- $S_{Hmax} = 1.6\ S_v$ in strike-slip faulting regions (assuming $S_V = (S_{Hmax} + S_{hmin})/2$,) and; (6b)
- $S_{Hmax} < S_v$ and $S_{hmin} = 0.6\ S_v$ in extension faulting regions. (6c)

Stress measurements have now been conducted in several deep boreholes to depths of almost 5 miles (8 km). Figure 4 shows that the measurements confirm the general stress gradients derived above. In particular, in the mid-plate compressive stress regions where these measurements were made, horizontal stresses well in excess of the vertical persist far down into the crust, and the horizontal stress gradient (k) is fairly consistent with a value ranging from approximately 1.3 to 2.

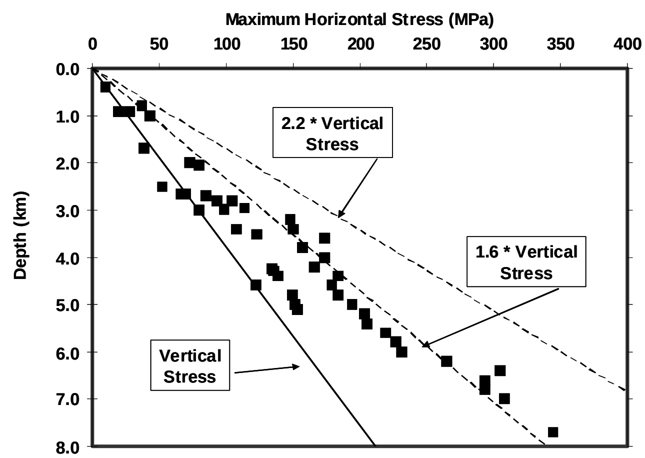

Figure 4. Stress measurements from boreholes deep in the earth's crust (data from Townend and Zoback, 2000).

Zoback and Zoback (2002) also state that they have found "no evidence" that "residual stresses" from past tectonic events play any role in today's stressfields. They speculate that if such stresses exist at all, they can only be important "in the upper few meters or tens of meters of the crust where the tectonic stresses are small."

If plate tectonics are responsible for virtually all the stresses measured at depth, and if the critically stressed crust model allows us to predict those stresses, then it is reasonable to expect that there is some relationship between the deep crustal stresses and those measured in coal mines. After all, the "near surface" is part of the crust! In fact, if we don't find a relationship, then we have a significant problem. For instance, if we conclude that the horizontal stress increases less rapidly than the vertical stress to depths of one or two thousand feet, but we know it exceeds the vertical stress at greater depths, the implication is that there is a major discontinuity in the stressfield somewhere. Let us then see what the actual measurements tell us.

[2] The discussion in this section is based almost entirely on the summary provided by Zoback and Zoback (2002).

OVERCORING STRESS MEASUREMENTS IN COAL MINES

Overcoring has been the most common technique for measuring stress in underground coal mines. In the US, most measurements have been made using the Bureau of Mines biaxial "borehole deformation gage" (Bickel, 1993). Internationally, the triaxial ANZI or CSIRO HI cells have been by far the most common (Mills, 1997; Nemcik et al., 2005; Cartwright, 1997). Most recently, downhole wireline stress measurement devices have been developed (Conover et al., 2004).

One important feature of overcoring stress measurements is that interpretation of the data requires the determination of the rock's elastic modulus (E). The modulus is not required for the other types of stress measurement contained in the WSM data base. Studies in the layered sedimentary geology of coal measure rock have found that the measured stresses in a single hole vary in proportion to the modulus of the rock (Aggson and Mouyard, 1988).

Cartwright (1997) pointed out that the relationships between horizontal stress and depth, like those of Hoek and Brown (1980), have typically displayed high scatter, particularly near the surface. Within his data base of UK stress measurements, there was a better correlation between stress and modulus than between stress and depth. He proposed that the two factors might be combined into a single equation:

$$S_H = B_0 + B_1 [(v/(1+v)) (\text{Depth})] + B_2 (\text{Modulus}) \qquad (7)$$

where B_0 is a constant with units of psi (MPa), B_1 is a constant with units of psi per ft of depth (MPa/m), v is Poisson's ratio, and B_2 is a dimensionless constant called the "tectonic strain factor" or TSF. Regression analysis provided the following values for the constants, with an r-squared of 0.94:

- B_0 = -580 psi (-4.0 MPa);
- B_1 = 0.4 psi/ft (0.009 MPa/m), and;
- $B_2 = 0.78*10^{-3}$

Cartwright's analysis indicated that for his data set, the modulus was more important than the depth for predicting the maximum horizontal stress.

Dolinar (2003) studied stress measurements from 37 eastern U.S. underground mines, including several stone mines. His analysis employed a version of equation (7), with B_0=0 and B_1 fixed at 1.1 psi/ft (0.025 MPa/m). He found that the remaining regression coefficient, the TSF (B_2), varied between 400 and 900 for the different geographic regions studied, with the highest TSF values in two small areas in central Appalachia.

Nemcik et al. (2005) also calculated the TSF for the large SCT data base of 235 measurements from Queensland and NSW mines. In their analyses, they also used equation (7), setting B_0=0 and B_1=1.1 psi/ft (0.025 MPa/m). In contrast to both Dolinar and Cartwright, however, Nemcik found that:

- There was a strong correlation between depth and the S_{Hmax} in both NSW and Qld,
- The TSF also increased significantly with depth, averaging 0.4 when the depth was less than 330 ft (100 m) deep, but more than 1.3 for mines at depths exceeding 1600 ft (500 m), and;
- At any given depth, a wide range of TSF values were measured.

Nemcik et al.'s work indicates that the TSF can vary significantly within a single region, and that the TSF cannot always explain a large proportion of the variation in S_{Hmax}.

INTERNATIONAL OVERCORING MEASUREMENTS DATA SET

If the magnitude of the near-surface stresses measured in coal mines were closely related to the deep-seated stresses measured by the WSM, we might expect to find that:

- The depth is at least as important as the modulus in predicting the horizontal stress, though both factors together should be better still;
- The depth gradient should be somewhere between 1.0-1.6 times the vertical stress for coalfields located in stable, a-seismic mid-plate areas, like those in the eastern U.S., the UK, Germany, or central Queensland;
- The depth gradient should be higher in a seismically active compressive regime like the one found in the Sydney Basin, and it should be lower an active extension regime like the one found in the western U.S. coalfields.

To test these hypotheses, a data set of 565 stress measurements was compiled. The heart of the data set is 373 measurements from underground coal mines. The breakdown of these by region is shown in figure 5. Approximately two-thirds of the coal data were from Australia, and were provided by SCT. Preliminary statistical analyses indicated that the four eastern U.S. coalfields could be combined into a single "eastern U.S. coal" grouping, and that the UK and German data could be combined into a "northern European coal" grouping.

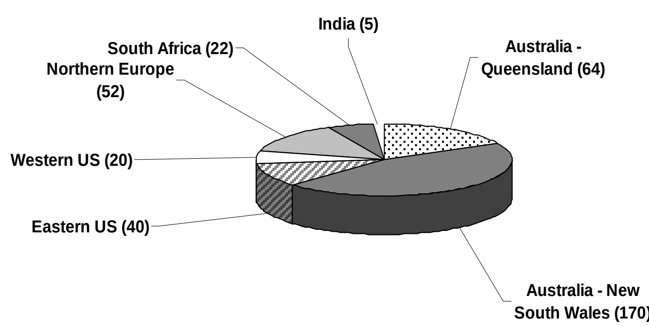

Figure 5. Locations of the stress measurements included in the coal data set.

In addition, about 200 non-coal measurements that were readily available in the literature and at the WSM website (Reinecker et al., 2005) were collected. The purpose of the non-coal data was to provide an independent check on the general regional trends observed within the coal data. The non-coal data set includes stress measurements from the same general regions as the coal data set, though it does not include any measurements from Australia. The non-coal data also provides an opportunity to compare stress trends within bedded coal measure strata to those in other geologic settings. However, it is recognized that while the coal data set is easily the most comprehensive of its kind ever compiled, the non-coal database is quite small compared to how many measurements could be available.

Preliminary analyses were conducted to see where data sets could be combined. It was determined that the measurements from Ireland, the UK, northern Europe, and Scandinavia could be combined into a single "All Europe" grouping. Similarly, data from the eastern U.S., Canada, and the western U.S. were combined into a "North America" grouping. It was surprising that while there were distinct differences between the western and eastern US coal data sets, the trends within the U.S. non-coal data did not seem to vary much by region. Figure 6 shows the number of data points within each regional grouping.

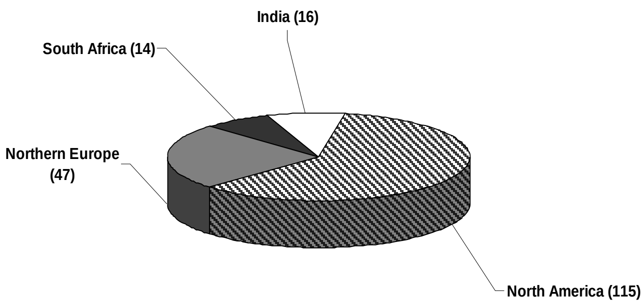

Figure 6. Locations of the stress measurements included in the non-coal data set.

Figures 7 and 8 show the range of depth and modulus within each regional group in the coal data set. The greatest depths are encountered in the western European mines (UK and Germany), while the highest modulus rocks are in the eastern U.S. Queensland has the lowest modulus rock. Note that the small size of the data set from India (n=5) and the relatively small range of depth in the South African data set mean that the results from these two regions would be expected to be less reliable than those from the other regions.

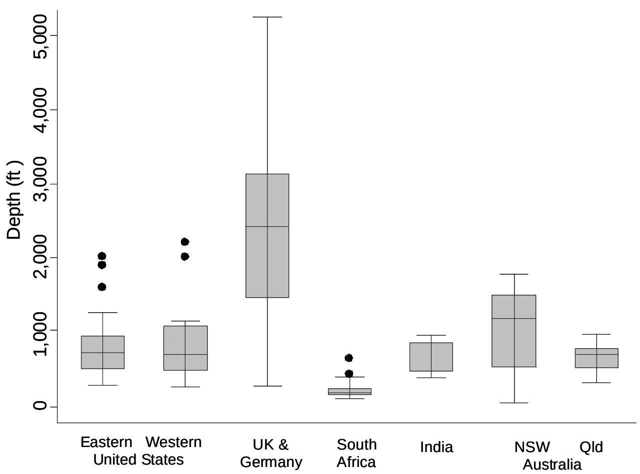

Figure 7. Range of depths of the coal stress measurements, by region. The middle line represents the median value, and the upper and lower hinges of the box represent the 75th and 25th percentiles of the data, and the dots are outliers.

The same types of summary data for the non-coal data set are shown in figures 9 and 10. The depth ranges are similar to coal data, except in South Africa where the non-coal data includes a number of measurements from extremely deep gold mines. In general, the modulus values for the non-coal measurements are about twice as great as those from the coal measurements.

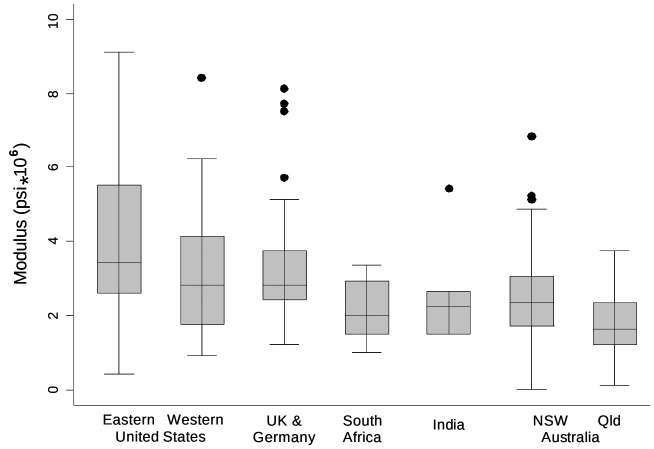

Figure 8. Range of elastic modulus of the coal stress measurements, by region.

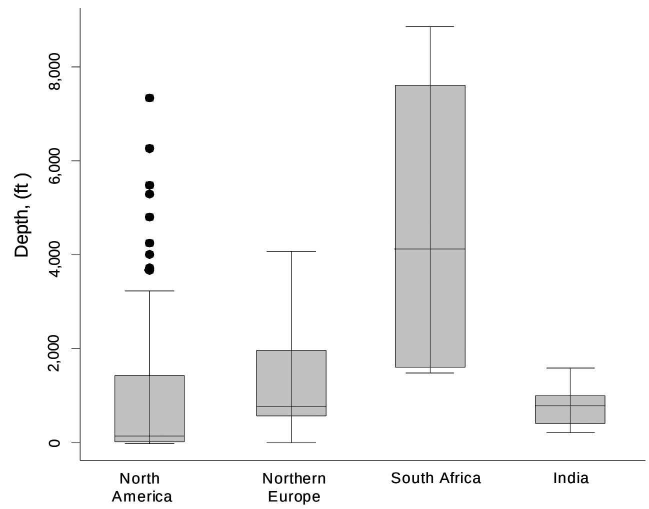

Figure 9. Range of depths of the non-coal stress measurements, by region.

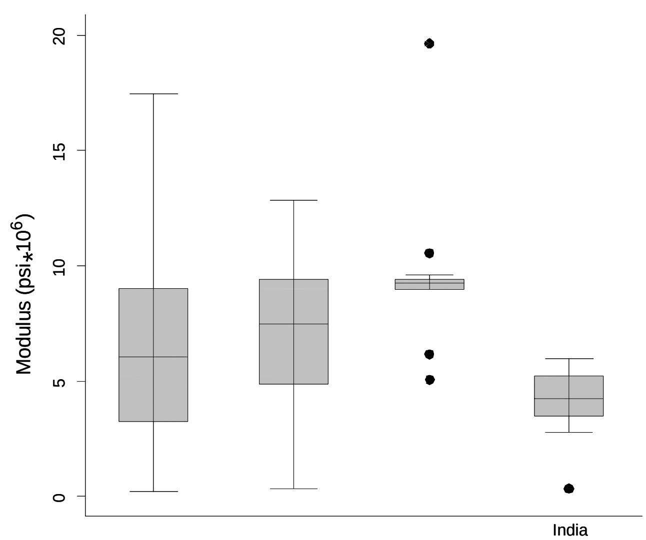

Figure 10. Range of elastic modulus of the non-coal stress measurements, by region.

Data Weighting: In order that the coal data not be overwhelmingly influenced by the Australian data, all the data was weighted by the following fomula:

$$\text{Weight of an individual measurement} = 1/ (n_R)^{0\,5} \qquad (8)$$

where n_R is the number of measurements from a particular region.

The result is that an individual data point from a region with few measurements will count more heavily than one from a region with a lot of measurements, but overall the heavily populated regions will still have more influence. For example, the data set contains 40 measurements from the eastern U.S. and slightly more than 4 times as many measurements from NSW. In the weighted analysis, an individual U.S. measurement is given a weight about two times as great as a measurement from NSW, but in aggregrate all the NSW measurement have twice as much influence over the final equation as do all the eastern U.S. measurements.

As a check, all of the analyses were run using both weighted and unweighted data, and it was found that the results did not differ significantly. The weighted results will be reported here in this paper.

Depth Gradients and Modulus Effect for the complete data sets: Multivariate regression was conducted using the statistical package STATA. The first analyses looked at the relationship between depth and stress, with no other variables. The form of the model is thus:

$$S_{Hmax} = B_0 + B_1 \text{ (Depth)} \qquad (9)$$

In this analysis, B_1 is the gradient of the maximum horizontal stress with depth (in psi/ft), and the intercept B_0 can be interpreted as the "excess stress" that is not associated with the depth gradient.

The results are shown in table 1:

Table 1: Regression results using Equation (9) as the model.

	Excess stress B_0 (psi (MPa))	**Depth gradient B_1 (psi/ft (MPa/m))**	**r-squared**
Coal data	1057 (7.3)	1.06 (0.024)	0.33
Non-coal data	1386 (9.6)	1.24 (0.028)	0.56
Combined data set	1159 (8.0)	1.18 (0.026)	0.50

In other words, for the combined data set, we can explain half of the variation simply by the depth (and a constant). All the other variability—regional location, modulus, proximity to the entry and other measurement errors (Gadde et al., 2006), is only responsible for the other 50%. Although depth is not as strong a predictor in for the coal data as it is for the non-coal data, the depth gradients are all also approximately equal to the vertical stress gradient of 1.1 psi/ft (0.025 MPa/m). The similarity between the coal and non-coal equations is striking—these are two completely independent data sets, drawn from similar parts of the world.

When the analyses were run with just the modulus, instead of depth, the r-squared values were all reduced. In the case of the coal data, the reduction is only from 0.33 to 0.28, but for the non-coal data (and the complete data set) the r-squared is reduced from about 0.5 to approximately 0.20. These results indicate that modulus effect is most pronounced in layered, coal measure geologies, but that even there depth explains about as much of the variation in the data as the modulus does.

In the next set of analyses, the effects of modulus and depth are explored simultaneously. The regression equation that is used is:

$$S_{Hmax} = B_0 + B_1 \text{ (Depth)} + B_2 \text{ (Modulus)} \qquad (10)$$

In this model, the excess stress consists of two components, the intercept (B_0) and plus the modulus term (B_2*E). (Note that with all these models, the excess stress is independent of the depth. Statistical analyses confirmed that there was minimal interaction between the modulus term and the depth within this data set.)

The results in Table 2 show that adding the "modulus factor" improves the r-squared values considerably, from 0.33 to 0.52 for the coal data, and to above 0.60 for the non-coal data set. Note that the values for the depth gradient drop slightly but are still close to 1.1 psi/ft (0.025 MPa/m). The biggest change from table 1, particularly for the non-coal data, is a reduction in the constant. In other words, it appears that the two elements of the excess stress, the intercept and the modulus term, are closely related.

Table 2. Regression results using equation (10) as the model.

	Intercept B_0 (psi (MPa))	**Depth gradient B_1 (psi/ft(MPa/m))**	**Modulus factor $B_2(10^{-3})$**	**r-squared**
Coal data	100 (0.7)	0.87 (0.020)	0.44	0.52
Non-coal data	284 (2.0)	1.16 (0.026)	0.19	0.62
Combined data	368 (2.6)	1.12 (0.025)	0.21	0.58

It is also worth pointing out that the relative importance of the depth effect and excess stress change with depth. For example, for a coal mine at 300 ft (90 m) of cover and a modulus of 3×10^6 psi (4.3 GPa), the total predicted horizontal stress is 1693 psi (11.9 MPa). The excess stress accounts for 1,432 psi (9.9 MPa), and the depth effect contributes just 261 psi (1.8 MPa) to this total. If the depth increases to 1,500 ft (450 m), the predicted stress is now 2737 psi (18.9 MPa), and all the increase is due to the depth effect.

Region-by-region analyses: The next step was to determine predictive equations in the form of equation (10) for each of the individual regions. It is reasonable to expect that different regions might have different relationships between stress, modulus, and depth, owing to different characteristics of the crust, tectonic forces, etc. The results of the individual analyses are shown in table 3.

Several observations can be made on the data in table 3:

- The r-squared for the individual equations are between about 0.50 and 0.70 for nine of the eleven data sets, which is not significantly better than what was achieved by analyzing the entire data set.

- The most consistent coefficients are the depth gradients (B_1), and these are also generally the most statistically significant coefficients. The range is from about 0.6 to 1.8 psi/ft (0.013 - 0.040 MPa/m) for the larger data sets.
- The modulus factors are also fairly consistent, ranging from about 250 to 600 x 10^{-3}. However, in four of the smaller data sets, these coefficients are not significant. Moreover, the modulus effect is more pronounced in the coal measure rocks.
- The excess stress, determined using the mean modulus value for each region, was remarkably consistent, averaging about 1,000 psi (7 MPa) for the eight largest data sets. The range was from a low of 400 psi (2.8 MPa) in Queensland coal mine measurements, to a high of 1,600 psi (11 MPa) in the eastern U.S. coal measurements.

Overall, while the equations for some regions (Western U.S., NSW) seem reasonably reliable, in many cases the small size of the individual data sets is probably responsible for poor correlations.

Unified analyses, controlling for different regional depth gradients: A statistical technique is available that allows us to combine the power of using the largest possible data set, while simultaneously allowing for some regional variation. This is accomplished by allowing the coefficients for the depth gradient to vary region by region[3].

The final equations are in the form of equation (10), but while all of them use the same coefficients for the components of the excess stress , each region has its own depth gradient. The results are shown in table 4.

Table 3. Regression results for regional subsets of the data, using equation (10) as the model.

	n	Intercept B_0 (psi (MPa))	Depth gradient B_1 (psi/ft (MPa/m))	Modulus factor $B_2(10^{-3})$	r-squared
East U.S. coal	42	369* (2.6)	1.34 (0.030)	0.302	0.55
West U.S. coal	20	-915 (-6.4)	0.66 (0.015)	0.62	0.71
UK/Ger coal	52	-249* (-1.7)	0.55 (0.012)	0.51	0.50
South Africa coal	22	866 (6.1)	-0.03* (0)	-0.01*	0
India coal	5	376* (2.6)	1.29 (0.029)	-0.04*	0.79
NSW coal	170	-633 (-4.4)	1.78 (0.040)	0.56	0.71
Qld coal	64	-210 (-1.5)	1.40 (0.031)	0.34	0.51
U.S./Can non-coal	115	-273 (-1.9)	1.45 (0.033)	0.27	0.70
N. Europe non-coal	47	905 (6.3)	1.24 (0.028)	0.02*	0.55
South Africa non-coal	14	327* (2.3)	2.03* (0.046)	-0.04*	0
India non-coal	16	-826 (-5.8)	2.20 (0.049)	0.26	0.56

*Signifies statistic is not significantly different from 0 at the 95% confidence level.

Table 4. Stress prediction parameters for equation 10 determined for the individual coal regions using the unified analysis regression technique.

	n	Intercept B_0 (psi (MPa))	Depth gradient B_1 (psi/ft (MPa/m))	Modulus factor $B_2(10^{-3})$
East U.S. coal	42	-298 (-2.1)	1.64 (0.037)	0.41
West U.S. coal	20	-298 (-2.1)	0.78 (0.018)	0.41
UK/Ger coal	52	-298 (-2.1)	0.71 (0.016)	0.41
South Africa coal	22	-298 (-2.1)	1.11 (0.025)	0.41
India coal	5	-298 (-2.1)	0.44 (0.010)	0.41
NSW coal	170	-298 (-2.1)	1.84 (0.041)	0.41
Qld coal	64	-298 (-2.1)	1.36 (0.031)	0.41

[3]The technique involves creating interaction terms involving dummy variables (Wooldridge, 2006, pp. 244-252). In this case, a dummy variable is defined based on region, and then that dummy variable is interacted with the variable "depth."

The regression analysis used to obtain table 4 achieves an r-squared of 0.69. Accounting for nearly 70% of all the variation in such a large and diverse data set is an impressive accomplishment. The following further observations can be made on these results:

- The greatest depth gradient, approaching 2.0, was found for the NSW measurements, which corresponds to the prediction based on the WSM;
- The results for the eastern U.S., Queensland, and the Western U.S. are also in good agreement with WSM predictions, and;
- The depth gradient for Europe is lower than was expected based on the WSM. Previous researchers have hypothesized that extensive past mining in the UK and German coalfields may account for the discrepancy (Muller, 1991).

The equivalent results for the non-coal data are shown in table 5. It is significant that all but one of the regional depth gradients determined for the non-coal data are very similar to the gradients found for the coal data. The exception is India where the coal data set is very small.

Table 5. Stress prediction parameters for equation 10 determined for the individual non-coal regions using the unified analysis regression technique.

	n	Depth gradient B_1 (psi/ft (MPa/m))	Modulus factor $B_2(10^{-3})$
U.S./Can non-coal	115	1.51 (0.034)	0.21
N. Europe non-coal	47	0.98 (0.022)	0.21
South Africa. non-coal	14	0.95 (0.021)	0.21
India non-coal	16	1.54 (0.035)	0.21

*The constant (B_0) was not statistically significant at the 95% level.

Figure 11 is a type of "residual plot" that helps evaluate the validity of the regression equation. It compares the maximum horizontal coal mine stresses predicted by table 4 with the measured ones for all regions except Australia. The Australian data appears on figure 12. Both figures show that the residuals (discrepancies) are quite evenly distributed for all regions and stress levels, which indicates that the regression results are valid.

Figure 13 plots the coal stress measurements against the depth of cover. Two predicted stress gradients derived from Table 4 are shown in the figure, in addition to the vertical stress gradient. The gradient for the "normal" grouping, which includes the eastern US, Australia, South Africa, and India, is based on an average gradient of 1.6 psi/ft (0.037 MPa/m) and an average modulus of $2.5*10^6$ psi (18 GPa). The "low gradient" grouping includes the western U.S. and the UK and German coalfields. The average stress gradient for this group is 0.93 psi/ft (0.021 MPa/m) and the average modulus is $3.2*10^6$ psi (22 GPa).

Analysis Using Modulus and Tectonic Strain Factor (TSF): A number of analyses were conducted using model represented in equation (7), and employing the TSF concept as defined by Dolinar (2003) and Nemcik et al. (2005). In these analyses, when only the modulus was used as the dependent variable, the r-squared values were not much different than those obtained when modulus was regressed against S_{Hmax} (less than 0.30 for the coal data, and less than 0.2 for the non-coal data). In an alternative analysis, individual TSF values were determined for each region. This analysis found the highest TSF in NSW, with a value of about 0.85, while the TSF determined for the Queensland, U.S., and European coalfields was about 0.45. The r-squared for this analysis was only 0.50, however, considerably lower than the 0.69 obtained with table 4.

Analysis of the minimum principal stress, S_{hmin}: No data was available from Australia for analysis of the minimum principal stress. The results of the analyses for the other regions are shown in Table 6. The model that was employed is shown in equation 11:

$$S_{hmin} = B_0 + B_1 \text{ (Depth)} + B_2 \text{ (Modulus)} \qquad (11)$$

The same "unified" regression technique used to obtain tables 4 and 5 was used in the analysis. The r-squared for the regression using the coal data was 0.54, and it was 0.67 for the non-coal data. The constant was not statistically significant in either equation.

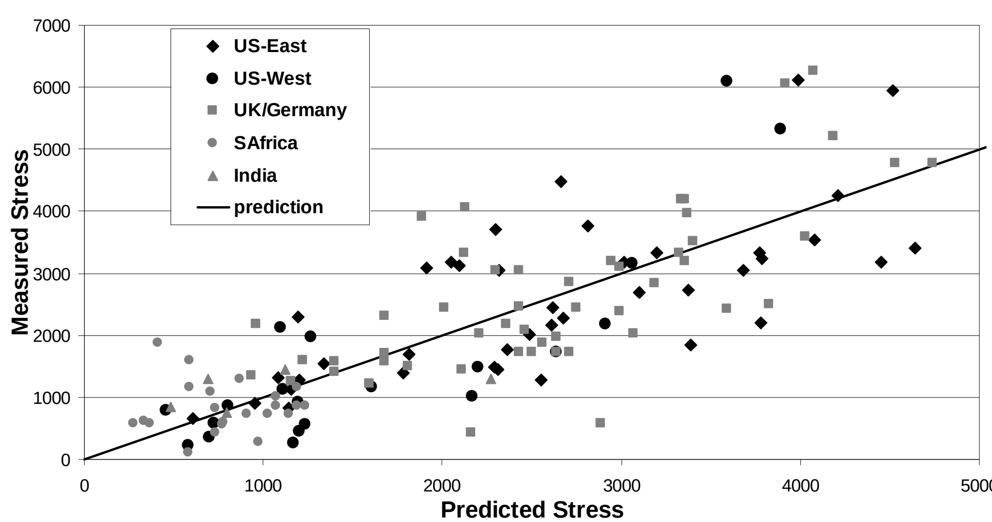

Figure 11. Residual plot for table 4, non-Australian data.

Table 6. Stress prediction parameters for equation 10 determined for the individual non-coal regions using the unified analysis regression technique.

	n	Depth gradient B_1 (psi/ft (MPa/m))	Modulus factor $B_2(10^{-3})$
East U.S. coal	42	1.34 (0.030)	0.15
West U.S. coal	20	0.56 (0.013)	0.15
UK/Ger coal	52	0.42 (0.009)	0.15
South Africa coal	22	0.20 (0.005)	0.15
India coal	5	0.42 (0.009)	0.15
U.S./Can non-coal	115	0.89 (0.020)	0.12
N. Europe Non-Coal	47	0.48 (0.011)	0.12
South Africa non-coal	14	0.44 (0.010)	0.12
India non-coal	16	1.03 (0.023)	0.12

DISCUSSION

At the beginning of the last section, several predictions were made about the horizontal stress measurements based on the WSM "critically stressed crust" model. In nearly every instance, the prediction was confirmed by the analysis:

- The depth was as important as the modulus in predicting the horizontal stress in the coal mine data set, and it was a much better predictor in the non-coal data set. When both factors were combined, the accuracy of the predictions improved significantly.
- The calculated depth gradient of 1.6 times the vertical stress for the eastern U.S., and 1.4 times the vertical stress in the Bowen Basin, was within the range of what was predicted for coalfields located in stable, a-seismic, mid-plate areas. The depth gradient that was calculated for the northern European coalfields of the UK and Germany was a little lower than expected, but even there it was still 0.9 times the vertical stress.

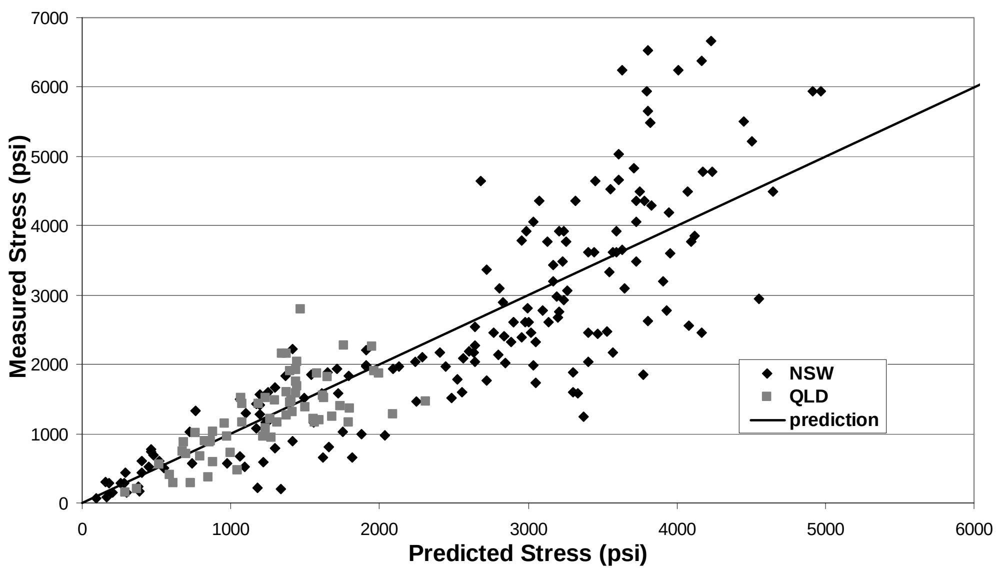

Figure 12. Residual plot for table 4, Australian data only.

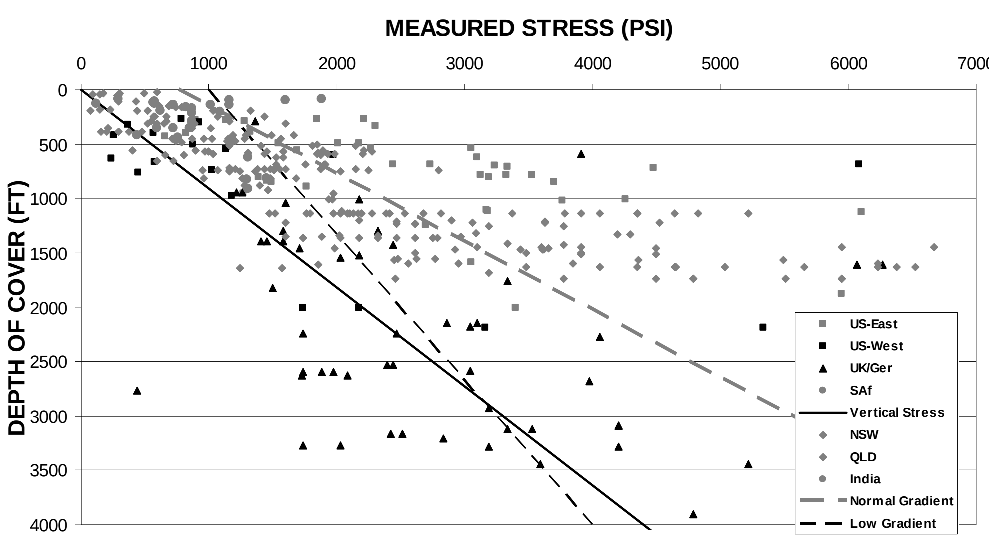

Figure 13. The coal stress measurements data base, showing predicted ranges of stress for regions with "normal" and "low" depth gradients (see text for more details).

- The greatest depth gradient was found to be in the seismically active compressive regime of the Sydney Basin, and one of the lowest depth gradients was in the active extension regime of the western U.S. coalfields.

It is reassuring that the depth gradients that were determined empirically seem to match up well with those that have been identified in the deep crustal measurements made by the WSM. The findings of this study indicate that we neither have to look for nor explain a discontinuity in stress at the transition between the "near surface" and the "deep" crust.

The study also confirmed that the "excess stress" is one significant component of the near-surface stress that does not fit directly into the WSM model. The excess stress typically adds 500-1,500 psi (3.5-10.5 MPa) to the horizontal stress that would be predicted by the depth gradient alone. At depths of less than 1,000 ft (300 m), the excess stress is usually responsible for at least 50% of the measured maximum horizontal stress.

Some important characteristics of the excess stress that were observed in the analysis are:

- Its magnitude, averaging about 1,000 psi (7 MPa), is quite constant over a broad range of independent data sets;
- Its magnitude does not noticeably increase with depth.
- The value of the modulus factor was about half as large in the non-coal data set as it was in the coal data set. However, since the average modulus in the coal data set is about twice the average non-coal modulus, the overall excess stress is about the same in both geologic regimes.
- The modulus factors determined from the analysis of the minimum principal stresses are consistently about half as large as those calculated for the maximum principal stress. In other words, the modulus effect is just as anisotropic as the vertical stress gradients.

Taken together, these observations indicate that the source of the excess stress is likely the same current tectonic regime that is responsible for the other aspects of today's stressfields, not "residual stress" or some other cause that would likely vary from locality to locality. One possible explanation is that while the "critically stressed crust" model normally assumes that the cohesion on fault planes is very small relative to the stresses at depth, near the surface that assumption may not be valid (Zoback et al., 2003). The cohesional strength of critically stressed faults may be enough to allow the crust to carry 1,000 psi (7 MPa) of stress more than would be the case if the cohesion actually was zero.

One further observation is that the modulus factor was more important to predicting the stress in the coal mine data set than it was in the non-coal data. In fact, for the coal data, the modulus was almost as powerful a predictor of the stress as the depth. It makes sense that in layered, sedimentary coal measure strata there would be a strong tendency for the stiffer rock units to carry more load. On the other hand, the horizontal strain within the rock is certainly not, in general, constant regardless of depth.

While the stress prediction equations that were developed during this study are quite robust statistically, it should be recognized that there are significant unexplained variability in the data. Greater variability would certainly be expected near the surface in mountainous terrain, where topographic effects are likely to be substantial. In situ stresses are also likely to be less predictable in seismically active coalfields like NSW and the western U.S.. Site specific stress measurements are still the only technique that can provide assurance of the local stress conditions.

A final comment is that there now seems to be no justification for employing the "Poisson's effect" or other static earth theories in any aspect of the analysis of in situ stress. The horizontal stresses measured underground are caused by the large plate tectonic forces that are currently being carried by the earth's crust. Poisson's effect can play a role when loads are actively applied to the ground, as in a longwall tailgate (Frith and Colwell, 2006), but there is no theoretical or empirical basis for using it to explain any aspect of the in situ stresses that develop over geologic time.

CONCLUSIONS

Using the largest data base of in situ stress measurements from coal mines ever assembled, this study found that the data fits the "critically stressed crust" model based on plate tectonics theory surprisingly well. Past studies had already shown that plate tectonics explained the ***orientation*** of the stresses observed underground, now it seems it explains their ***magnitudes*** as well. In particular, the maximum horizontal stress was found to increase with depth at rates of approximately 0.8-2.0 times the vertical stress. The study also identified a second component of the measured stress, called the "excess stress," which adds approximately 500-1,500 psi (3.5-10.5 MPa) to the total. The excess stress is apparently independent of the depth.

The study makes a major contribution by providing equations for estimating the maximum horizontal stress in several major coalfields around the world. More importantly, the study provides a framework for understanding the source of the horizontal stresses encountered underground every day. By showing that these "near surface" stresses are tied to those found in the deep crust, it links the worlds of rock engineering and geophysics. Hopefully, future research can build on this foundation to develop better tools for understanding and predicting in situ stress, and for using those predictions to design safer underground structures.

Disclaimer

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